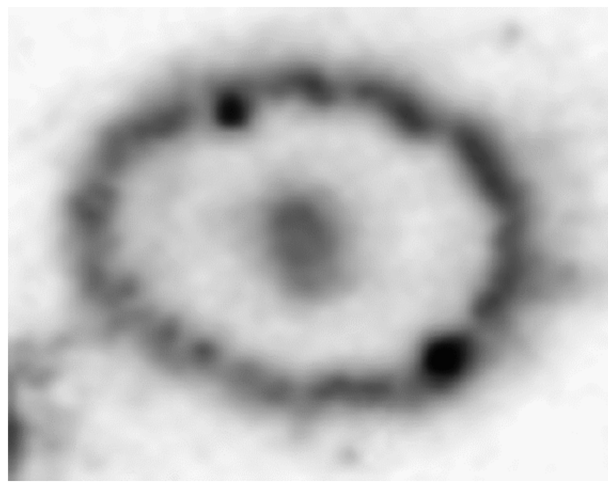
Astro 501, Fundamental Astronomy Problem Set 2

Due Sept 16

1. As long as we're on the subject of geometric distances and luminosities, there's one more method that I haven't talked about – light echos. The most famous analysis is that for the for supernova SN 1987A. Let's do the problem.

In 1987, a supernova went off in a LMC, which is a satellite galaxy of the Milky Way. (It was the first naked-eye supernova since the one discovered by Kepler!) At first, the spectrum of the supernova showed normal absorption/emission lines associated with gas moving outward at $\sim 10,000$ km s^{-1}, but 83 days after the explosion, a series of narrow (< 10 km s^{-1}) emission lines were detected with the International Ultraviolet Explorer (IUE) satellite. (IUE had no imaging capability, but it could take spectra.) The strengths of these lines brightened over the next few months, until day 413, when they reached a plateau.

Two years later, when the *Hubble Space Telescope* was launched, an image of the supernova revealed the presence of an elliptical ring (semi-major axis of $0\overset{''}{.}83$) surrounding the still-unresolved supernova remnant. This ring was likely *not* part of the supernova ejecta: it was probably there long before the supernova went off (from, perhaps, the mass-loss of a previous stage of stellar evolution).



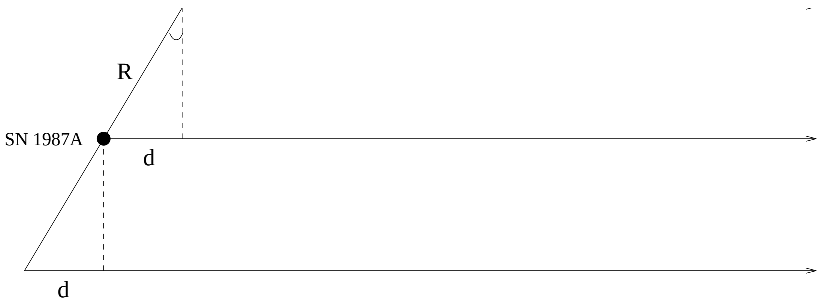
Now assume that a) the ring is actually a circular (not spherical) shell of material inclined to the plane of the sky, and the narrow emission lines cited above were produced by the supernova's light interacting with (i.e., "echoing" off) the shell.

a) Draw a picture illustrating the geometry of the system, and explain why it took 83 days for the emission lines to show up, and another 330 days for the emission lines to reach their peak.

b) From the emission line data alone, derive the inclination of the ring. Is this number consistent with the imaging data from *HST?*

c) What is the distance to the supernova, and therefore the Large Magellanic Cloud?

This is the geometry of the problem.



It took light 83 days to reach the ring. The subsequent brightening of the ring is due to the difference in light travel time from the ring to the observer.

The path difference between light coming directly from the supernova and light echoing off the near side of the ring is

$$\Delta = R - d = R - R\sin i = R\,(1 - \sin i)$$

Therefore the brightening began where the near side ring's light was observed

$$t_i - t_0 = \frac{R}{c}\,(1 - \sin i)$$

and continued until the light echoing off the far side of the ring reached us.

$$t_{\max} - t_0 = \frac{R}{c}\,(1 + \sin i)$$

Which implies

$$\sin i = \frac{t_{\max} - t_i}{t_{\max} + t_i}$$

Plugging in the numbers gives $i = 42^\circ$. This is consistent with the $\sim 43^\circ$ number you get from measuring the ring's major and minor axis.

Since the inclination of the ring is now known, we can now derive R

$$t_{\max} - t_0 = \frac{R}{c}\,(1 + \sin i)$$

This gives us a radius for the ring of 0.2 pc. Since the projected radius is $0.''83$, the true distance is 51 kpc, or $(m - M)_0 = 18.55$.

2. For strong lines, the equivalent width of an absorption feature goes approximately as the number of absorbing atoms to the two-fifth's power. Combine the Saha equation with the Boltzmann distribution to show how the strength of Hα (6563 Å) should change as a function of temperature (in the range $4,000 < T < 50,000$) for a stellar atmosphere with electron pressure $\log P_e = 1$ (dynes/cm^2). To create this plot, you will need the following information:

- The ionization potential of hydrogen is 13.6 eV from the ground state; the energy difference between the ground state of hydrogen and the first excited state ($n = 2$) is 10.2 eV.

- The statistical weight of the $n = 2$ state of hydrogen is 4.

- The partition function of ionized hydrogen $u_{II} = 1$.

- The partition function of neutral hydrogen depends on the temperature parameter, $\Theta = 5040/T$ in the following manner:

Θ	0.07	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.70
$\log u_H$	4.43	3.94	3.19	2.42	1.06	0.37	0.30	0.30	0.30

The problem assumes that the stellar atmosphere under consideration is in Local Thermodynamic Equilibrium (LTE). To calculate the number of atoms in the $n = 2$ state of hydrogen, you first need to know the fraction of hydrogen atoms that are neutral (as opposed to ionized). This is given by the Saha equation

$$\frac{N(HII)}{N(HI)} N_e = \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} \frac{2u_{II}}{u_H(T)} e^{-\chi/kT}$$

where $\chi = 13.6$ eV is the ionization potential of hydrogen and $u_H(T)$ and u_{II} are the partition functions of neutral and ionized hydrogen, respectively. If you substitute in the ideal gas law for electrons, i.e.,

$$P_e = N_e kT$$

and let f_1 equal the fraction of neutral hydrogen atoms, then the Saha equation becomes

$$\log \frac{1 - f_1}{f_1} = \log \frac{N(HII)}{N(HI)} = -\left(\frac{5040}{T}\right)\chi - \log P_e + 2.5 \log T - 0.48 + \log \frac{2u_{II}}{u_H(T)}$$

Next, we need to know the fraction of neutral hydrogen atoms that are in the $n = 2$ state. The fraction in $n = 2$, compared to $n = 1$ is given by the Boltzmann distribution

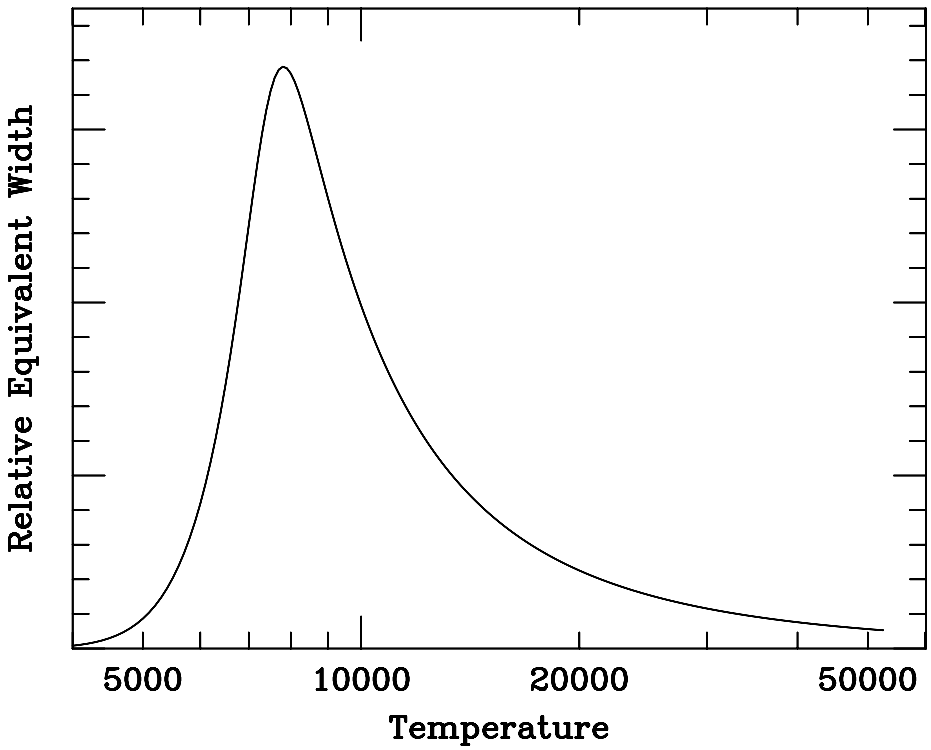
$$\frac{N(n = 2)}{N(n = 1)} = \frac{\omega_2}{\omega_1} e^{-\chi_{1,2}/kT}$$

where $\chi_{1,2}$ is the energy difference between the first and second energy levels. The ratio of the number of atoms in the $n = 2$ state to the number of atoms in all states is

$$f_2 = \frac{N_i}{N(HI)} = \frac{\omega_i}{\omega_0 e^{+\chi_{i0}/kT} + \omega_1 e^{+\chi_{i1}/kT} + \omega_2 e^{+\chi_{i2}/kT} + \ldots}$$

$$= \frac{\omega_i e^{-\chi_i/kT}}{\omega_0 + \omega_1 e^{-\chi_1/kT} + \omega_2 e^{-\chi_2/kT} + \ldots} = \omega_i \frac{e^{-\chi_i/kT}}{u_H}$$

where $i = 2$, $\chi_2 = 10.2$ eV is the energy of the $n = 2$ state, and u_H is again the partition function. The strength of the absorption is then EW $=(f_1\, f_2)^{2/5}$. Below is the plot for $\log P_e = 1$.



3. The Sun has an absolute magnitude of $V = 4.83$. Use the table of parallax precision versus magnitude given in the notes to estimate the percent error versus distance that Gaia will eventually produce for solar-type stars. Plot the results over the range 100 pc to 2 kpc. If the space density of G main-sequence stars is ~ 0.004 pc^{-3}, how many solar-type stars can Gaia measure to an accuracy better than of 5%? (Just for fun: do you think this estimate is reasonably accurate? Why or why not?)

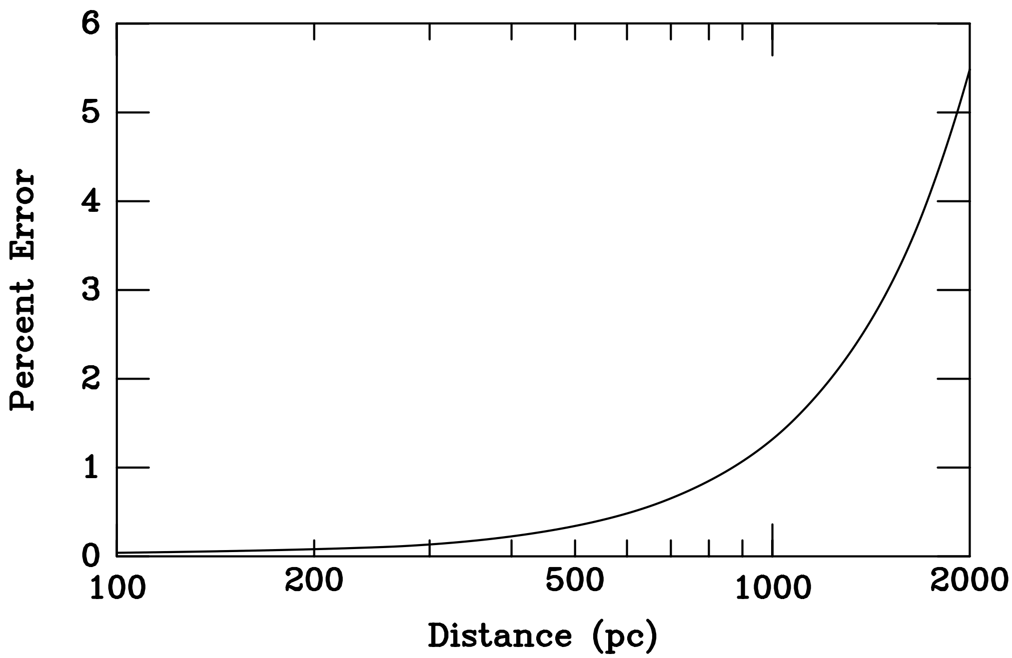
The parallax of object, π is related to its distance (in pc) by

$$d = 1/\pi$$

and the apparent magnitude of a star, as a function of distance is

$$m = M + 5\log d - 5$$

Interpolating in the table of errors then gives $\sigma(\pi)$, and the fractional error is $\frac{\sigma(\pi)}{\pi} = \sigma(\pi)\, d$. The error versus distance is shown below.



From the figure, Gaia can obtain 5% distances out to 1.9 kpc, i.e., over volume of 28.7×10^9 pc^3. If the density of G-type stars is 0.004 pc^{-3}, then there are $\sim 115 \times 10^6$ solar-type stars within range of Gaia.

Of course, this estimate assumes spherical symmetry, i.e., the density of stars is equal (and constant) in all directions. That's clearly not true: the Milky Way is a disk galaxy, and so the number density of stars falls off sharply perpendicular to the disk. So this number is an overestimate.